

# **Assimilation of Synthetic-Aperture Radar Data into Navy Wave Prediction Models**

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## **LONG-TERM GOAL**

To develop methods utilizing synthetic aperture radar (SAR) data to improve predictions for the littoral zone obtained from Navy wave forecasting models — in this case the SWAN model of Booij, Ris & Holthuijsen (1999).

## **OBJECTIVES**

There are three basic objectives to this program: (1) to develop a forward prediction capability for the expected value of the SAR-image spectrum, with the SWAN wave-spectrum prediction as input; (2) to develop methods to bring the SWAN-based SAR-spectrum prediction into agreement with satellite-based SAR observations by adjusting the SWAN model inputs; and (3) to validate the improvement in the results from the SWAN model against ground-truth data.

## **APPROACH**

The accuracy of the predictions obtained from the SWAN model is limited by (among other things) inaccuracies in the specification of the model inputs (initial conditions, boundary conditions, and forcing). The information contained in SAR images of ocean waves can potentially be used to improve the predictions, by guiding modifications to the model inputs so as to obtain the best agreement between the observed SAR-image spectrum and the SAR-image spectrum predicted from the wave spectrum output from the SWAN model.

For the calculation of the SAR-image spectrum corresponding to a given SWAN prediction of the wave spectrum, the approaches to be used are the fully nonlinear mapping approach of Hasselmann & Hasselmann (1991). The approach used to bring the predicted SAR-image spectrum into agreement with the observed SAR-image spectrum will rely on the variational framework outlined by Le Dimet & Talagrand (1986) which corresponds to the strong-constraint formalism of Bennett (1992). This approach requires the derivation of the adjoint of both the SWAN model and the SAR spectrum model. The adjoint model equations are forced by the difference between the predicted and observed SAR-image spectra, and their solution yields a 'correction' to the model inputs. This procedure is iterative in nature and is applied until the difference is minimized. Validation of the procedure will rely on comparison of model predictions both with and without SAR data assimilation to ground truth data in order to quantify the improvements.

## **WORK COMPLETED**

During FY 99, adjoints for both the Hasselmann & Hasselmann SAR model and the SWAN model were derived and implemented.

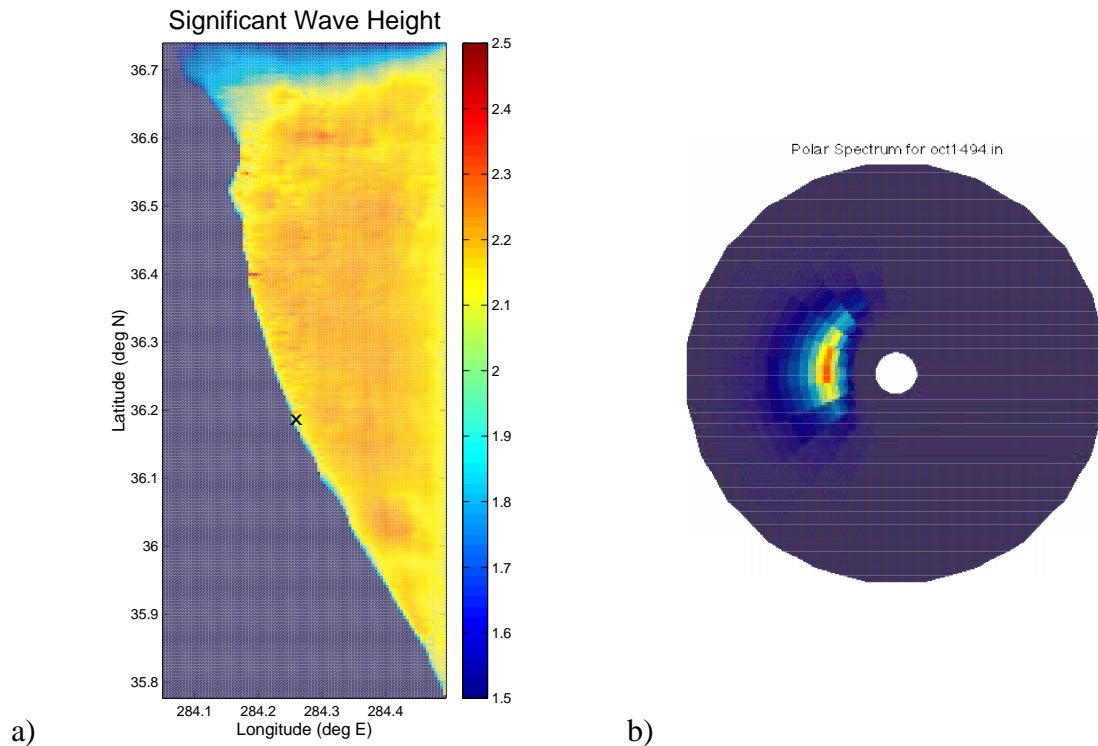


Figure 1 a) SWAN-predicted significant wave height for an October 1994 storm at the Duck FRF, the **x** indicates the location of the FRF 8 m array. (b) Incident wave spectrum, constant along eastern boundary.

## RESULTS

The main accomplishments of the current fiscal year are the implementation and testing of adjoint models for both the Hasselmann & Hasselmann SAR spectrum model, and the SWAN model. These adjoints have been tested independently as a prelude to combining them into a complete SAR assimilation procedure for the SWAN model. The SWAN adjoint has been tested using wave spectrum observations, using the assimilation procedure to determine the incident wave spectrum which produces predicted wave spectra which are consistent with observations at some number of spatial locations. The SAR adjoint has been tested in an inversion procedure designed to obtain the wave spectrum which results in the closest agreement between the predicted and observed SAR spectrum, with no *a priori* constraints being placed on the wave spectrum. The results of the independent tests are discussed below.

### Inverting the SWAN Model

Using the approach described in Le Dimet & Talagrand (1986) and Bennett (1992) a formal adjoint to the SWAN model was developed. The resulting equations are similar in nature to the SWAN model equations; i.e. transport equations including spatial and spectral advection with source terms. The source terms include ones corresponding to those in the SWAN model (wind input, white-capping, bottom friction, depth-induced breaking, and nonlinear wave-wave interactions) and an additional term proportional to the difference between wave spectrum observation and a previous SWAN prediction for some number of locations. Modifications were made to the SWAN code to allow solution of the adjoint form of the equations, along with appropriate I/O modifications and memory management to allow the observations to be input into the code. A user-selectable option was introduced into the SWAN code to choose either solution of the standard equations or the adjoint equations. The current version of the SWAN adjoint neglects the nonlinear source terms, and has only been tested for stationary conditions.

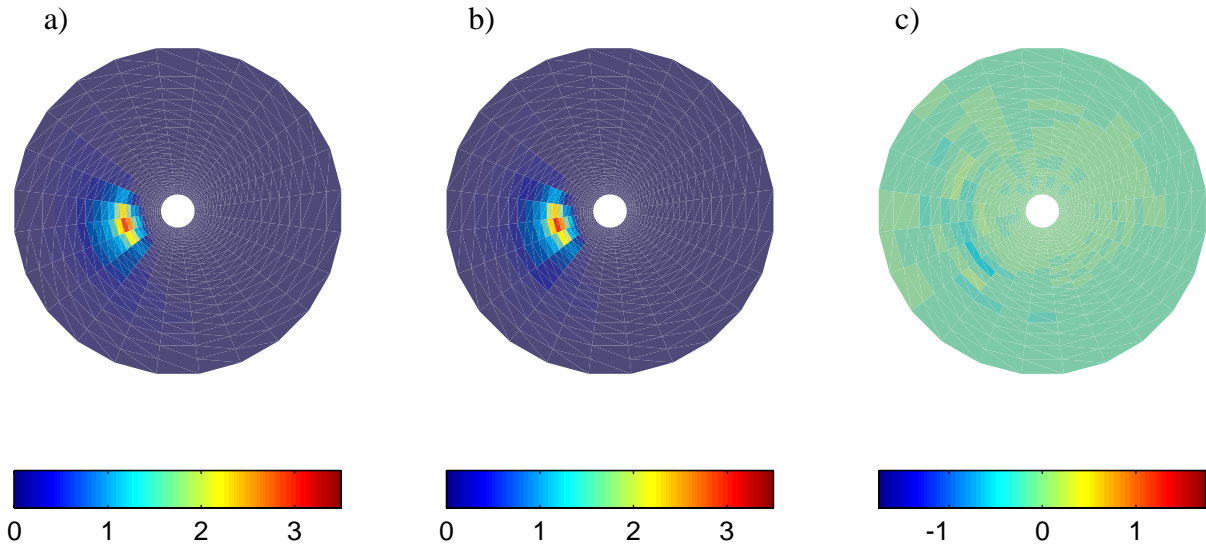


Figure 2 Assimilation results after five iterations and a cost function reduction of 98.9%:  
a) 'observed' spectrum at the FRF 8 m array location (from SWAN simulation); b) Estimated spectrum at the FRF 8 m array location; c) difference between a) and b).

To test the adjoint SWAN implementation, the following problem has been examined. Given wave spectrum observations for some number of spatial locations in a geographical region, determine the incident wave spectrum, and the corresponding prediction of the wave spectra for the entire region, which are consistent with the observations. The specific test implemented was for the region near the USACE Field Research Facility (FRF) in Duck, NC. A forward prediction using a known input spectrum was run using the SWAN model. The predicted wave spectrum for a given location or locations were extracted and used as 'observations' (in the example discussed below, a single spectrum from the location of the FRF 8 m array was chosen). The observations were then used as input to the adjoint SWAN model. The solution of the SWAN adjoint at the open boundary indicates the adjustment to the incident wave spectrum required to improve agreement between the predicted spectrum and the observations.

Figure 1 shows the incident wave spectrum and the predicted significant wave height for one of the test cases. In this case, the observations consisted of the wave spectrum at one location, that of the FRF 8 m array. The location of the FRF 8 m array is indicated by an **x** on the figure. Figure 2 shows the 'observed' spectrum at the 8 m array location, along with the estimated wave spectrum for that location obtained by inverting the SWAN model via the assimilation process. The estimated spectrum was obtained after five iterations of the forward/adjoint model pair, starting from an assumed zero spectrum as the initial guess. At this point the error variance between the prediction and observation has been reduced by 98.9% from that for the initial guess. A similar level of agreement is seen in the estimated incident wave spectrum and other quantities such as the significant wave height.

### Inverting the SAR Spectrum Model

The adjoint SAR model computes the difference between the observed SAR image spectrum and the predicted image spectrum, based on the current estimate of the wave spectrum, and calculates the direction in which the wave spectrum should be changed in order to reduce this difference. The result will be incorporated as a source term in the adjoint SWAN model. To verify the adjoint SAR model it has been exercised in a stand-alone mode in order to determine whether the wave spectrum could be estimated directly from the SAR image.

For the stand-alone adjoint SAR model, the wave height spectrum  $S(\vec{k})$  is changed at each iteration by the amount

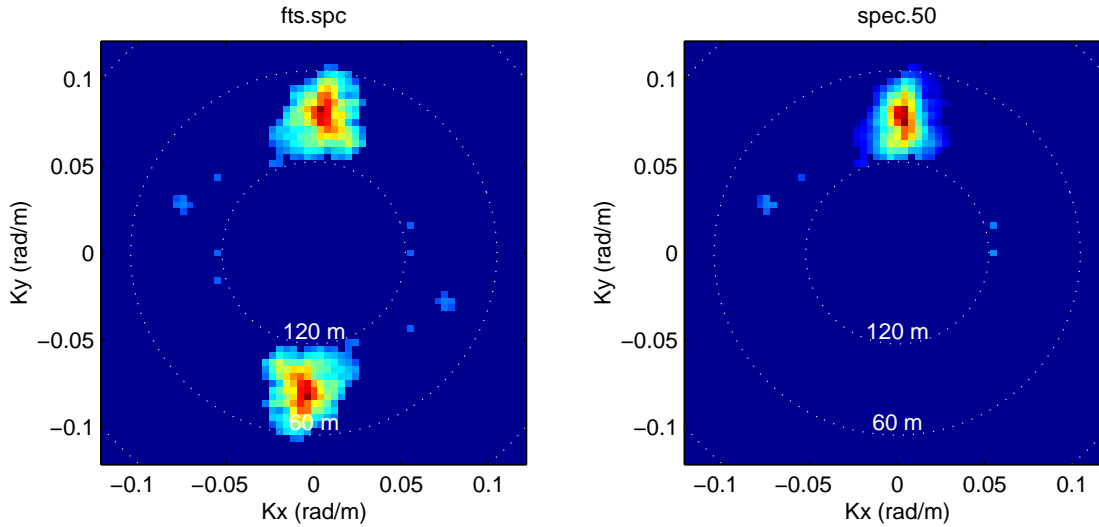


Figure 3. Results of SAR spectrum inversion using ERS SAR data (© ESA 1994) from 14 Oct 1994 near Duck, NC: a) Observed SAR image spectrum, after filtering and thresholding; b) Estimated wave height spectrum after 50 iterations.

$$\delta S(\bar{k}) = \varepsilon [S_{im}(\bar{k}) - \hat{S}_i(\bar{k})] [R_{sar}(\bar{k}) + R_{sar}(-\bar{k})]$$

where  $S_{im}(\bar{k})$  is the observed image spectrum,  $\hat{S}_i(\bar{k})$  is the predicted image spectrum using the current estimate of the wave spectrum, and  $R_{sar}(\bar{k})$  is the quasi-linear SAR transfer function. This expression is a first approximation to the full adjoint SAR model. The assimilation process was initiated using zero as the first estimate of the wave spectrum.

Tests were done using an actual ERS SAR image collected over Duck, North Carolina on 14 October 1994, and also using simulated SAR data. Figure 3(a) shows the observed image spectrum for a region near Duck, NC, at a water depth of approximately 16 m. This spectrum was obtained by Fourier transforming the image intensity over four 128x128 pixel subsets, averaging the squared magnitudes of these Fourier transforms, smoothing with a 3x3 boxcar filter, subtracting the spectral level due to speckle noise, and thresholding the spectrum at 20% of the peak value to remove low-frequency noise in the spectrum. Figure 3(b) shows the estimated wave height spectrum after 50 iterations, at which point the cost function had been reduced to about 1.5% of its initial value. The peak spectral density in the retrieved spectrum is roughly equal to that of the spectrum measured by the FRF 8-meter array, but the width is smaller than the measured spectrum, with the result that the significant wave height is smaller than the measured value by about a factor of two. Tests using simulated SAR spectra did not show a similar bias in the estimated wave spectrum. Therefore, the bias using actual ERS data may be due to a scaling problem between the model and the actual data or to losses incurred during preprocessing of the SAR spectrum.

## IMPACT/APPLICATION

Achieving the overall objectives of the program will result in an improved prediction capability for near-shore waves, allowing readily available remote sensing data to be used effectively.

## TRANSITIONS

As the data assimilation capability for the SWAN model is developed during the coming FY, its use may be extended beyond the assimilation of SAR data, to other data types by other participants in the BE effort.

## RELATED PROJECTS

This project is related to other efforts under the BE program. Improvements to the SWAN model (in both physical and numerical modeling) being pursued at NRLSSC, NPGS and WES will be included in the assimilation scheme as they are developed.

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